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The Influence of Local Winds on Fallout

Report No. 1

Contract DA 36-039 AMC-03283(E)

DA Project 3A 99-27-005

First Progress Report

1 July 1963 - 31 December 1963

Object: The object of this research is to study local wind systems in relation to their effect on distributions of fallout.

Prepared by:

Pieter J. Feteris
Edwin Kessler III
Edward A. Newburg

THE TRAVELERS RESEARCH CENTER, INC.

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PURPOSE

The purpose of the project is to increase understanding of the roles of local circulations in shaping patterns of atmospheric fallout at the earth's surface.

ABSTRACT

Model local circulations are discussed with regard to their influence on the disposition of airborne particles in a non-precipitating, non-diffusive atmosphere. Two-dimensional vertical circulations like the sea breeze along a straight coast, or like the circulations near mountain ranges, when steady-state, principally affect the time of establishment of a pattern and not its shape or the ultimate patterns of the total accumulations at the ground. When such calculations intensify or decay during the passage of fallout through them, both the duration and onset time of fallout at ground locations can be affected, with associated local changes of total accumulation.

Three-dimensional local circulations affect the place of deposit of airborne particles and the time of their deposition. Circulations confined to the horizontal plane can affect the place of deposit and shape of a fallout pattern, but vertical air currents must occur to change the time of its establishment.

Three-dimensional divergence accompanying vertical currents in the real compressible atmosphere significantly increases local concentrations in only those cases where updrafts are about equal to particle fall speeds; in such cases, the role of diffusion should be simultaneously considered.

The influence of local circulations, in any case, is greatest with respect to the smallest (slowest falling) particles. ( )

PUBLICATIONS, LECTURES, REPORTS, AND CONFERENCES

On August 13, 1963, Drs. E. Kessler III and E. J. Aubert, and Mr. P. J. Feteris visited USAERDL and discussed the work of this contract with Messrs. Walter Conover, Marvin Lowenthal, and Joseph Walsh. On October 8 and 9, Dr. Helmut Weickmann visited The Travelers Research Center, Inc., and discussed the work of the contract and future plans with Drs. Aubert and Kessler, and Mr. Feteris.

1.0 FACTUAL DATA

1.1 Introduction

The distributions of close-in fallout resulting from atomic explosions have usually been considered from the viewpoint of the descent of size-distributed particles in the temporally uniform wind field defined by radiowind reports [1, 2, 3, 4, 5]. Effects of widespread and localized precipitation have been considered by some investigators, notably Austin [6] and Greenfield [7]. Although the importance of local wind circulations has been recognized, especially in the case of high-yield bursts, there has been little treatment of kinematic processes fundamental to the effects of local circulations on fallout distributions. The present study is intended to fill this gap, and analyzes model circulations to provide understanding essential for prediction of the effects of local circulations and fallout in the real world.

Like Kellogg et al. [3], consider the distribution of contaminants in a stabilized cloud that exists as stem and mushroom some minutes after a nuclear detonation. The contamination products are assumed to move horizontally with the environmental wind u, while falling at speed V_i relative to the air. V_i, a function of height, is a negative number when it refers to the fall velocity.

Figure 1 represents a portion of the stem cloud within which particles of various diameters and fall speeds are distributed. If, for example, the wind speed increases at a constant rate with height, the trajectories of particles of fall speed V_i starting at the three points A_1, B_1, C_1 are like the parabolic arcs shown. Since the environmental wind field is assumed steady, all particles of fall speed V_i following trajectories which pass through points A_2, B_2, and C_2, respectively, keep to the same respective paths beyond the location of the initial cloud. It is appropriate then to consider a suitably large number of points on the leeward side of the stem cloud at the initial time; and, for each point, the corresponding mass δM_i of fallout material which falls at speed V_i and is contained in a stream tube of unit horizontal cross section, extending from the leeward points to the windward boundary or top of the cloud. Since the wind field in this first instance is nondivergent, the stream tube of unit horizontal-cross-section passes through each point with unaltered cross section to the ground.

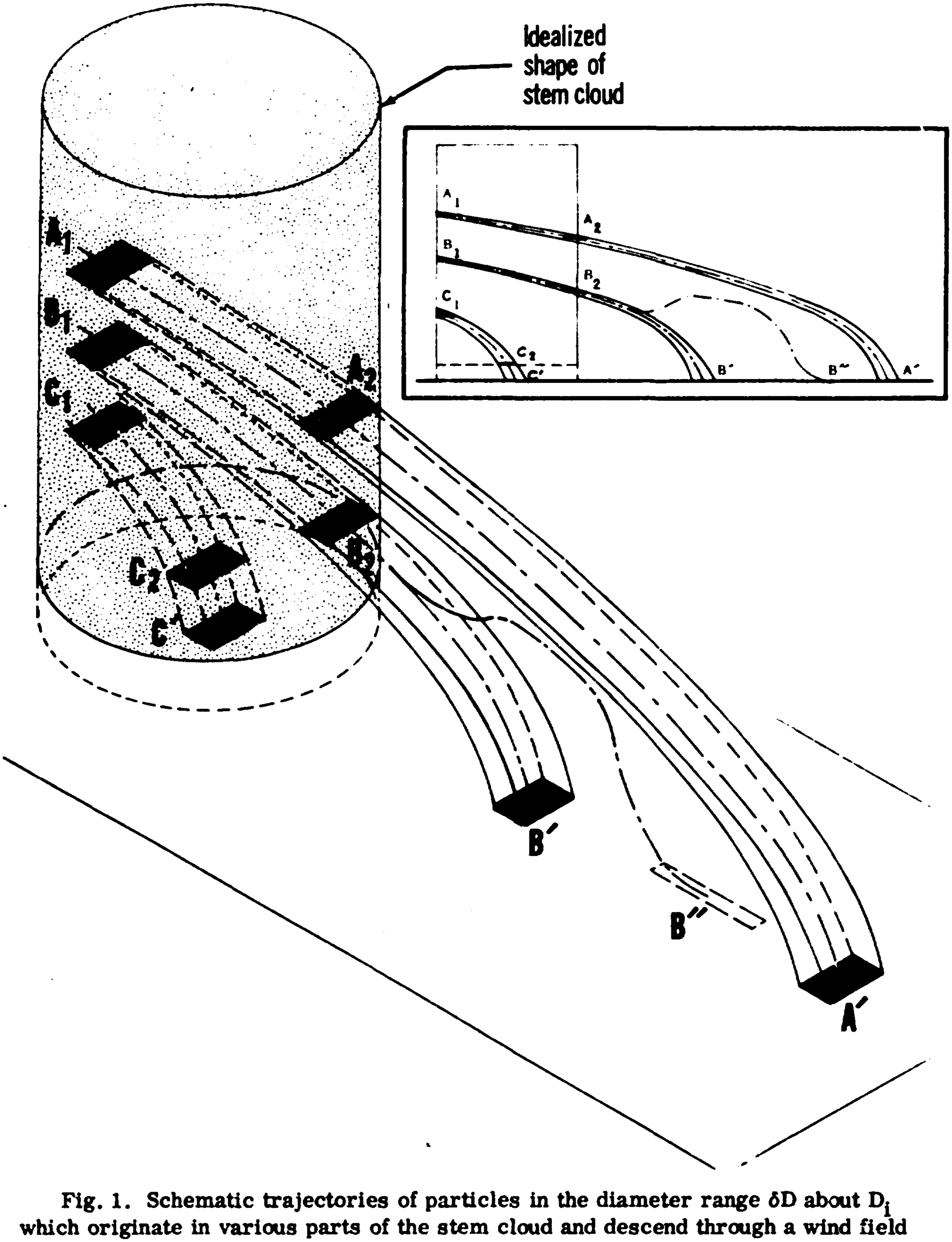

Fig. 1. Schematic trajectories of particles in the diameter range δD about D_i which originate in various parts of the stem cloud and descend through a wind field whose horizontal speed increases linearly with height without change of direction.

The mass per unit area associated with each point can be represented by the equation

$$\delta M_i = \int_s C_i \sin \Theta \, ds = \int_s C_i \, dz \tag{1}$$

where C_i is the concentration of material of fall speed V_i at each point along the trajectory within the cloud at the initial time, $\sin \Theta = -V_i/(V_i^2 + u^2)^{1/2}$, ds is incremental arc length along the trajectory, and dz is the height increment. Subscript s denotes that the integration takes place along the trajectory.

For each point, there is a corresponding mass of material, and we now have to define when and where it reaches the ground. Obviously, the first particles along a particular trajectory in the figure reach the ground when

$$T_{first} = - \int_0^{z_0} \frac{dz}{V_i}, \tag{2}$$

and the last when

$$T_{last} = - \int_0^{z_1} \frac{dz}{V_i}. \tag{3}$$

In Fig. 1, where Z_2 and Z_1 are the heights at which the trajectory intersects respectively the right and left hand boundaries of the atomic cloud. The duration of fallout is

$$T_{duration} = - \int_{z_0}^{z_1} \frac{dz}{V_i} \tag{4}$$

The negative signs in Eqs. (2), (3), and (4) correspond to our practice of representing the fall velocity V_i by a negative number. While the particles are descending, they are displaced to a horizontal distance X by the horizontal wind:

$$X = \int_0^T u dt = \int_z^0 \frac{u}{V_i} \quad . \tag{5}$$

Therefore, when the horizontal winds within the stabilized atomic cloud are defined, any point on the cloud edge can be associated with: a mass per unit area of fallout of fall speeds near V_i; a point on the ground; and (since the heights of the end points of the trajectory within the atomic cloud are defined) the times of termination of fallout at the ground (see Fig. 1). Isopleths drawn

to fit the values attached to points at the ground define the pattern of fallout due to particles falling in the range of speeds δV_i about V_i. The addition of patterns pertaining to calculations for particles of different fall speeds furnishes the time-dependent pattern of total fallout concentration at the ground. The isopleths of mass can be converted to levels of radioactivity from a relationship expressing the radioactivity of unit mass, of the form

$$R = R_0 e^{-kt}, \tag{6}$$

where R_0 and k depend on the nature of the nuclear detonation.

Methods developed by various investigators differ somewhat in their detailed assumptions and lead to forecast fallout patterns which are different from each other and from observations in the real world [4]. Consideration of the effects of local circulations may suggest means for improving the existing fallout models.

This report discusses how local circulations outside the atomic cloud can alter the mass of fallout, the time of onset, and points on the ground associated with points on the boundary of the stabilized cloud. The treatment assumes as above that the diffusion is unimportant, though cases occur in which the omission of diffusion must lead to relatively large errors. The study uses idealized models to facilitate mathematical treatment and the development of physical understanding.

1.2 Continuity Equations for Airborne Particles

Consider air moving at a velocity having components in the x, y, and z directions of u, v, and w respectively (w is positive in the upward direction). The air contains particles of concentration C, which fall relative to the air at the velocity V_i, a negative quantity. Then a development similar to Haurwitz [8] gives the following fundamental continuity equation for the concentration of particles during their descent towards the surface.

$$\frac{\partial C}{\partial t} = - \left\{\frac{\partial}{\partial x} (Cu) + \frac{\partial}{\partial y} (Cv) + \frac{\partial}{\partial z} [C(w + V_i)]\right\} \tag{7a}$$

Equation (7a) assumes that particles whose concentration is C are neither created nor destroyed on their way down and that they are distributed homogeneously

over a volume of air. A similar continuity equation holds for the motion of the air

$$\frac{\partial \rho}{\partial t} = - \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w). \tag{7b}$$

In the atmosphere it is practically always true that horizontal changes and local time changes of the air density are relatively small. By omitting these terms, Eq. (7b) can be simplified to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = - w \frac{\partial \ln \rho}{\partial z}. \tag{8}$$

Substitution of (7b) into (7a) yields

$$\frac{\partial C}{\partial t} = - u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} - (w + V_i) \frac{\partial C}{\partial z} - C \left[\frac{\partial V_i}{\partial z} - w \frac{\partial \ln \rho}{\partial z}\right], \tag{9}$$

c.f. Kessler [9]. The derivatives of the wind appearing in Eq. (7a) are absent from Eq. (9); the density term which replaces them accounts for the compressibility of the atmosphere.

Equation (9) defines the change in concentration at a fixed place in the air in terms of the winds and fall speeds at the place and the gradients of the concentration near the place. For many purposes, however, it is convenient to be able to refer to changes of concentration following the motion of particles. A local change can always be equated to the sum of individual and advective changes, i.e.,

$$\frac{\partial C}{\partial t} = \frac{dC}{dt} - u \frac{\partial C}{\partial x} - v \frac{\partial C}{\partial y} - (w + V_i) \frac{\partial C}{\partial z}. \tag{10}$$

Substitution of Eq. (10) into Eq. (9) yields

$$\frac{d \ln C}{dt} = - \frac{\partial V_i}{\partial z} + w \frac{\partial \ln \rho}{\partial z}. \tag{11}$$

This equation is a cornerstone. It says that only two factors (diffusion omitted) affect the concentration of the set of fallout particles falling at speed V_i. One factor is the vertical variation of the fall velocity V_i; the other is the atmosphere's compressibility (i.e., three-dimensional divergence of vertically moving air). Since the fall velocity of a particle varies principally as a result of its entering denser air, both terms in Eq. (11) can be related to the compressibility.

1.3 The Effects of Steady Vertical Air Circulations in an Incompressible Atmosphere

Since the terms on the right side of Eq. (11) usually have magnitudes of the order 10^{-4} sec^{-1}, the concentration of airborne fallout is not importantly affected by them over the course of an hour or so. In an incompressible atmosphere, the fall speed of a particle relative to the air is constant and Eq. (11) becomes

$$\frac{d \ln C}{dt} = 0; \qquad C = \text{constant.} \tag{12}$$

Equation (12) is accurate for circulations whose depths are a few kilometers or less, and where vertical currents are considerably smaller than the fall speeds of the particles.

1.3.1 Descent of Particles Through Vertical Air Currents Without Horizontal Motion

It is obvious that particles falling through steady vertical currents, where the horizontal air motion is zero, arrive at the ground at the same location that would have been their destination in the absence of air currents. The effect of vertical currents on fallout patterns is to change the time when they are established. The time T_1 for particles with fall velocity V_i to descend through a layer of depth H without a vertical circulation is $T_1 = -(H/V_i)$. When there is a vertical circulation, the time is approximately $T_2 = -[H/(V_i + \bar{w})]$.

If the updraft w in the vertical column anywhere exceeds V_i, particles cannot reach the ground. When updrafts exceed fall speeds, however, other factors become important. These include diffusion, changes of the horizontal wind field which carry the particles out of the updraft column, and the effects of three-dimensional divergence due to the compressibility of the atmosphere.

The time taken for particles of various fall speeds to fall through vertical currents defined by (see Kessler [9])

$$\frac{dz}{dt} = w = \frac{4w_{max}}{H} \left(z - \frac{z^2}{H}\right) \tag{13}$$

can be found by integrating the sum of this expression with the fall speed between the upper boundaries of the circulation and the surface:

$$t = \int_H^0 \left[\frac{4w_{max}}{H}\left(z - \frac{z^2}{H}\right) + V_i\right]^{-1} dz. \tag{14}$$

The time taken for a particle to travel through the updraft core of the circulation is

$$t_{up} = \frac{H}{w_{max}\left(-\frac{V_i}{w_{max}} - 1\right)^{1/2}}\left[\tan^{-1}\frac{1}{\left(-\frac{V_i}{w_{max}} - 1\right)^{1/2}}\right] \tag{15}$$

The travel time through the downdraft is given by

$$t_{down} = \frac{H}{4|w|_{max}\left(-\frac{V_i}{|w|_{max}} + 1\right)^{1/2}}\left\{\ln\frac{2\left[1 + \left(1 - \frac{V_i}{|w|_{max}}\right)^{1/2}\right] - \frac{V_i}{|w|_{max}}}{2\left[1 - \left(1 - \frac{V_i}{|w|_{max}}\right)^{1/2}\right] - \frac{V_i}{|w|_{max}}}\right\} \tag{16}$$

in which $|w|_{max}$ is the maximum absolute value of the vertical velocity.

Figure 2 shows the travel times in relation to the particle fall speeds and the vertical velocities of the air.

1.3.2 Role of Complete Cellular Circulations with Vertical Air Currents

Of course the presence of vertical currents implies the existence of horizontal divergence and horizontal air velocities. The updraft and downdraft columns treated in the previous section occur at particular horizontal positions in a circulation depicted in Fig. 3. If w_{max} defines the maximum vertical velocity, $z = 0$ and $z = H$ the lower and upper boundaries of this two-dimensional circulation and L its width, the conditions:

a) $\partial u/\partial x + \partial w/\partial z = 0$

b) $w = 0$ for $z = 0$ and $z = H$

c) $w = w_{max}$ for $z = H/2$

are satisfied by equations of the following type

$$u = \frac{2Lw_{max}}{\pi H}\left(\frac{2z}{H} - 1\right)\sin\frac{2\pi x}{L} \tag{17a}$$

$$w = \frac{4w_{max}}{H}\left(z - \frac{z^2}{H}\right)\cos\frac{2\pi x}{L} \tag{18a}$$

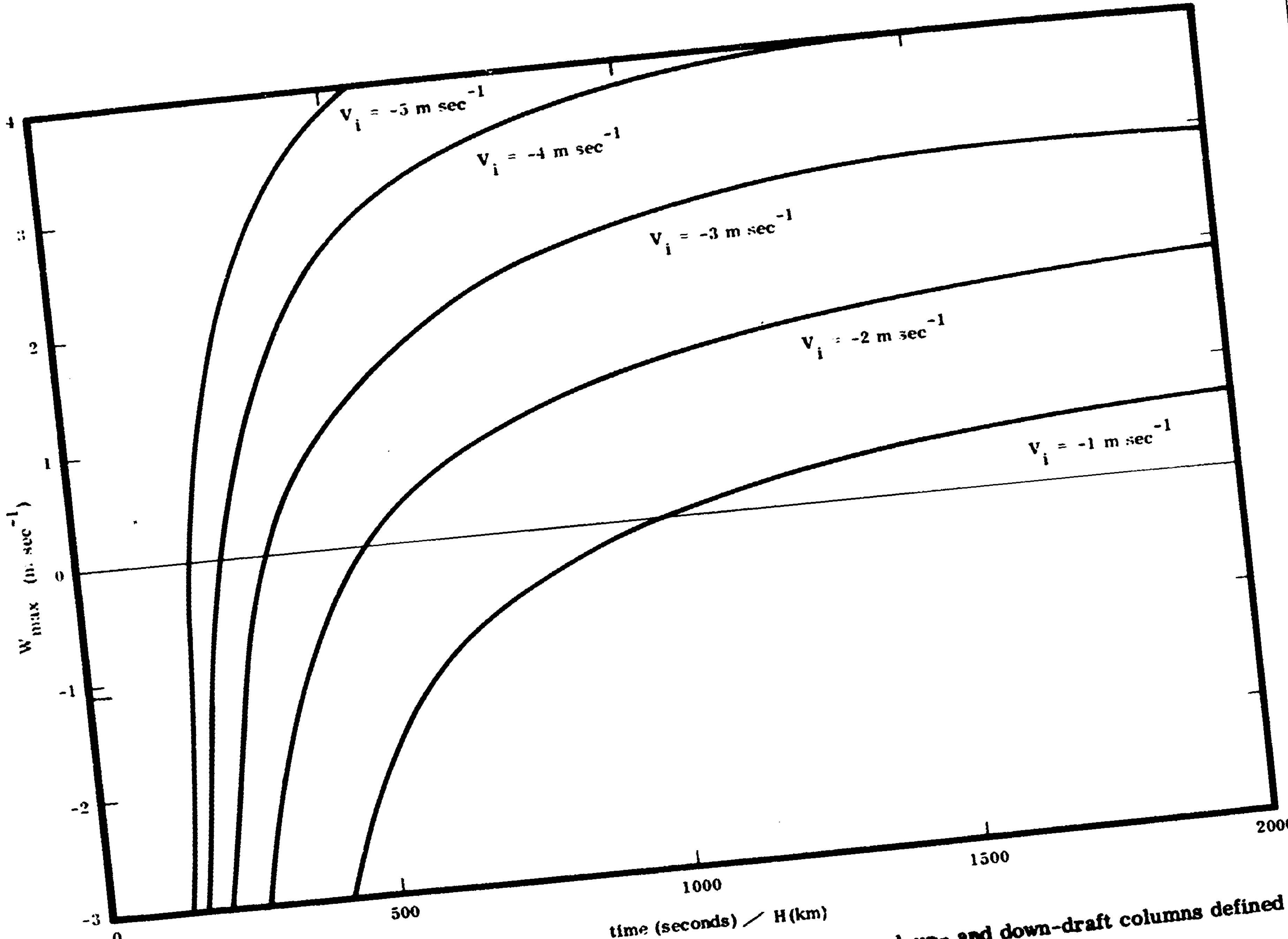

Fig. 2. Arrival times of fallout with different fallspeeds descending through up- and down-draft columns defined by Eq. (13).

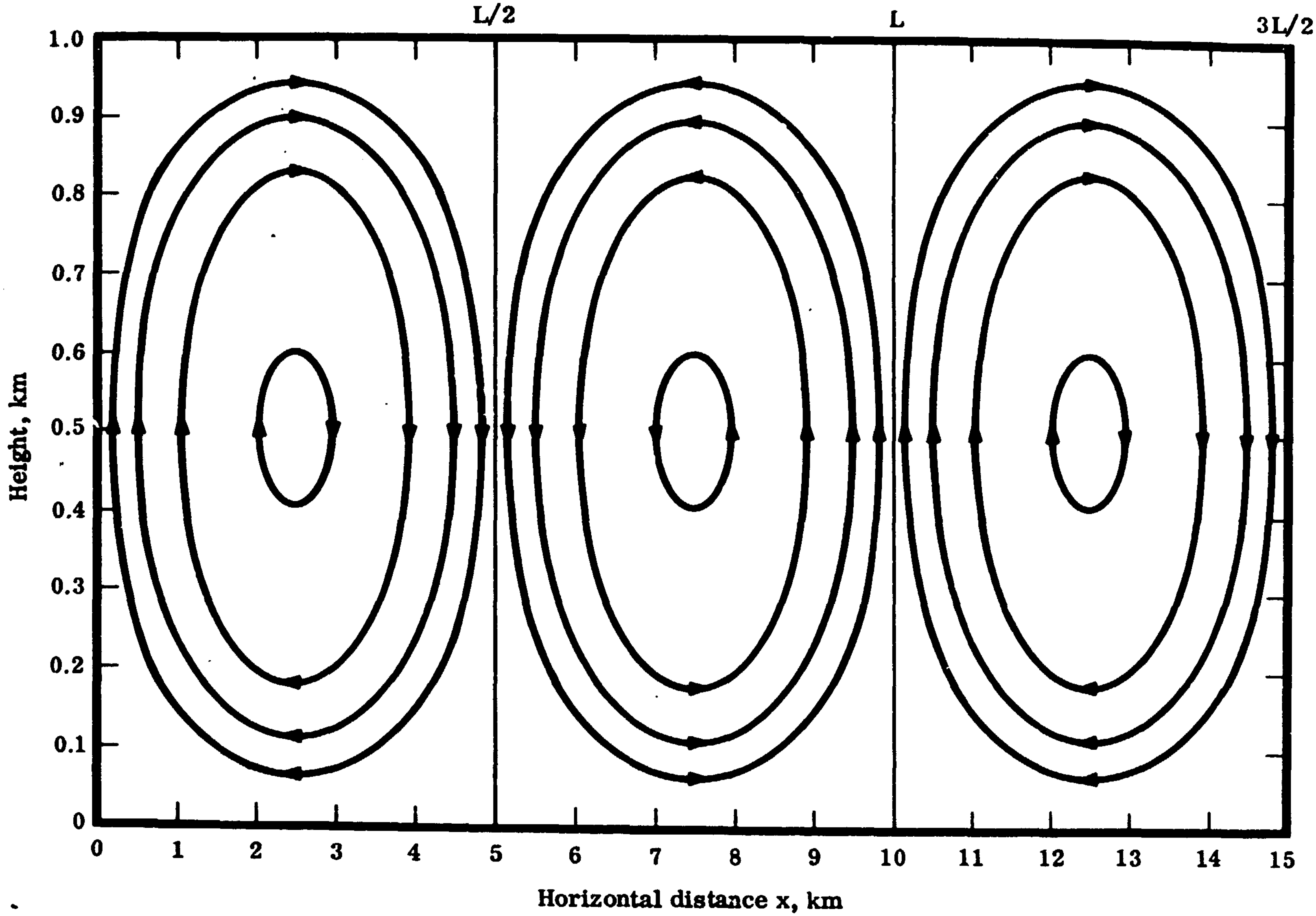

Fig. 3. Local circulation given by the stream function

$$\psi = \frac{2Lw_{max}}{\pi H}\left(z - \frac{z^2}{H}\right) \sin \frac{2\pi x}{L}.$$

for the motion of air.

With addition of an ambient windfield $u_0 = f(z)$, Eq. (17a) becomes

$$\frac{\partial x}{\partial t} = \frac{2Lw_{max}}{\pi H}\left(\frac{2z}{H} - 1\right) \sin \frac{2\pi x}{L} + [f(z)]. \tag{17b}$$

For particles descending through the circulation with a fall speed V_i, Eq. (18a) becomes

$$\frac{\partial z}{\partial t} = \frac{4w_{max}}{H}\left(z - \frac{z^2}{H}\right) \cos \frac{2\pi x}{L} + V_i . \tag{18b}$$

It is possible to represent the trajectories of the particles by a stream function ψ, which satisfies $\partial u/\partial x + \partial(w + V_i)/\partial z = 0$, by putting

$$\frac{\partial x}{\partial t} = - \frac{\partial \psi}{\partial z} \quad \text{and} \quad (w + V_i) = \frac{\partial \psi}{\partial x}.$$

If this stream function indeed exists, the following equation must be satisfied:

$$\psi = - \int \frac{\partial x}{\partial t} dz = \int (w + V_i)\, dx. \tag{19a}$$

Now

$$- \int \frac{\partial x}{\partial t} dz = \frac{2Lw_{max}}{\pi H}\left(z - \frac{z^2}{H}\right) \sin \frac{2\pi x}{L} - \int f(z)\, dz + f(z) \tag{19b}$$

and

$$\int (w + V_i) dz = \frac{4w_{max}}{\pi H}\left(z - \frac{z^2}{H}\right) \frac{L}{2\pi} \sin \frac{2\pi x}{L} + V_i x + f(z). \tag{19c}$$

Equations (19b) and (19c) are equal if

$$\psi = \frac{2Lw_{max}}{\pi H}\left(z - \frac{z^2}{H}\right) \sin \frac{2\pi x}{L} + V_i x - \int f(z)\, dz. \tag{19d}$$

In this case, the fall velocity V_i is assumed to be constant. However, if V_i is a function of z it will not be possible to derive ψ.

It is instructive to plot the streamlines of fallout descending at constant speed V_i through the circulation defined by Eqs. (17b), (18b), and (19d) with $f(z) = 0$. Figure 4 shows the streamlines when $-(V_i/w_{max}) = 2$. Note that each streamline of fallout terminates at a point directly beneath the point where it enters the circulation. In other words, this local circulation does not affect the pattern of concentration at the ground, but only the time of its establishment. Sediment descending through updrafts arrives later than would otherwise be the

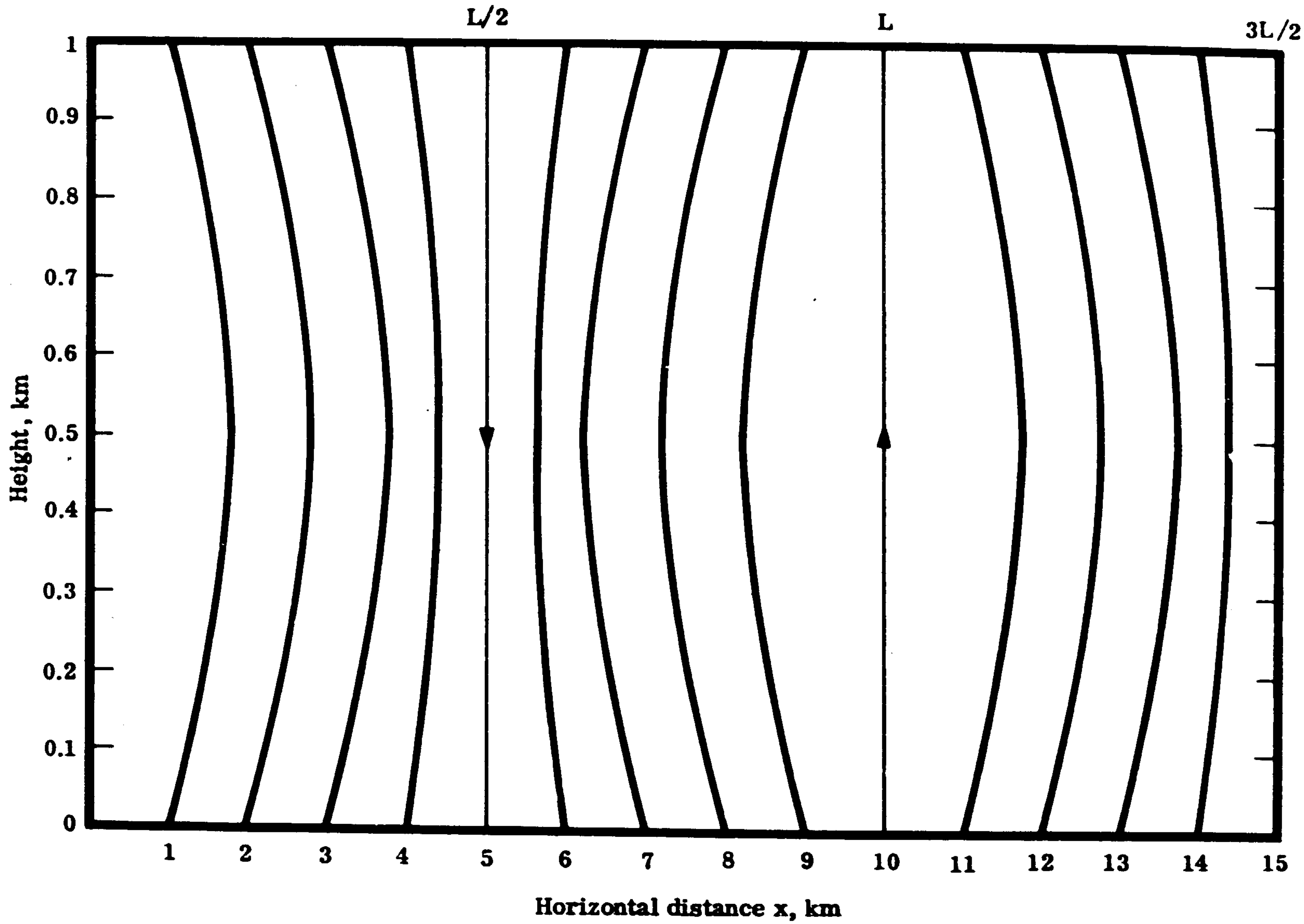

Fig. 4. Trajectories of fallout descending through the local circulation pictured in Fig. 3, where $-V_i/w_{max} = 2$.

case, and that descending through downdrafts arrives earlier. At $x = L/2$ and $x = L$ (see upper boundary of Fig. 4), the arrival times are given by Fig. 2, and elsewhere there is a smooth variation between these extremes.

That the locations of the exit points of fallout streamlines are unchanged by any steady local circulations where $V_i = 0$ and the incompressible approximation applies [i.e., cases described by Eqs. (17b), (18b), and (19d)] is readily seen by considering implications of the converse. If the exit points were changed, the termini of some of the streamlines would be comparatively crowded; elsewhere comparatively separated. This implies a change of the concentration C. But Eq. (12) says that C remains unchanged. Figures 5a, 5b, 6a, and 6b show that the entrances and exit points of streamlines of fallout are also unchanged by the presence of a local circulation in an atmosphere with ambient shear.

The invariance of the fallout pattern can also be proved mathematically as follows. The local wind field under consideration satisfies the continuity condition

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0 \tag{20}$$

and, by definition, the horizontal winds vanish at the horizontal limits of the local circulation; the vertical wind at the vertical limits

$$u\ (0, z) = u\ (L, z) = 0$$

and

$$w\ (x, 0) = w\ (x, H) = 0. \tag{21}$$

A local wind field can be the center section of Fig. 3; a sea-breeze-like local circulation model is shown in Fig. 7. A particle entering a region R where $0 \leq z \leq H$ follows a path determined by

$$\frac{dx}{dz} = \frac{u + f(z)}{w + V_i} \tag{22}$$

where V_i is the constant fall speed of particles of the i^{th} class, and f(z) is the environmental horizontal wind, i.e., the u-component at the horizontal limits of the local circulation. The continuity condition implies the existence of a stream function $\psi(x, z)$ such that

$$\frac{\partial \psi}{\partial x} = w; \qquad \frac{\partial \psi}{\partial z} = - u. \tag{23}$$

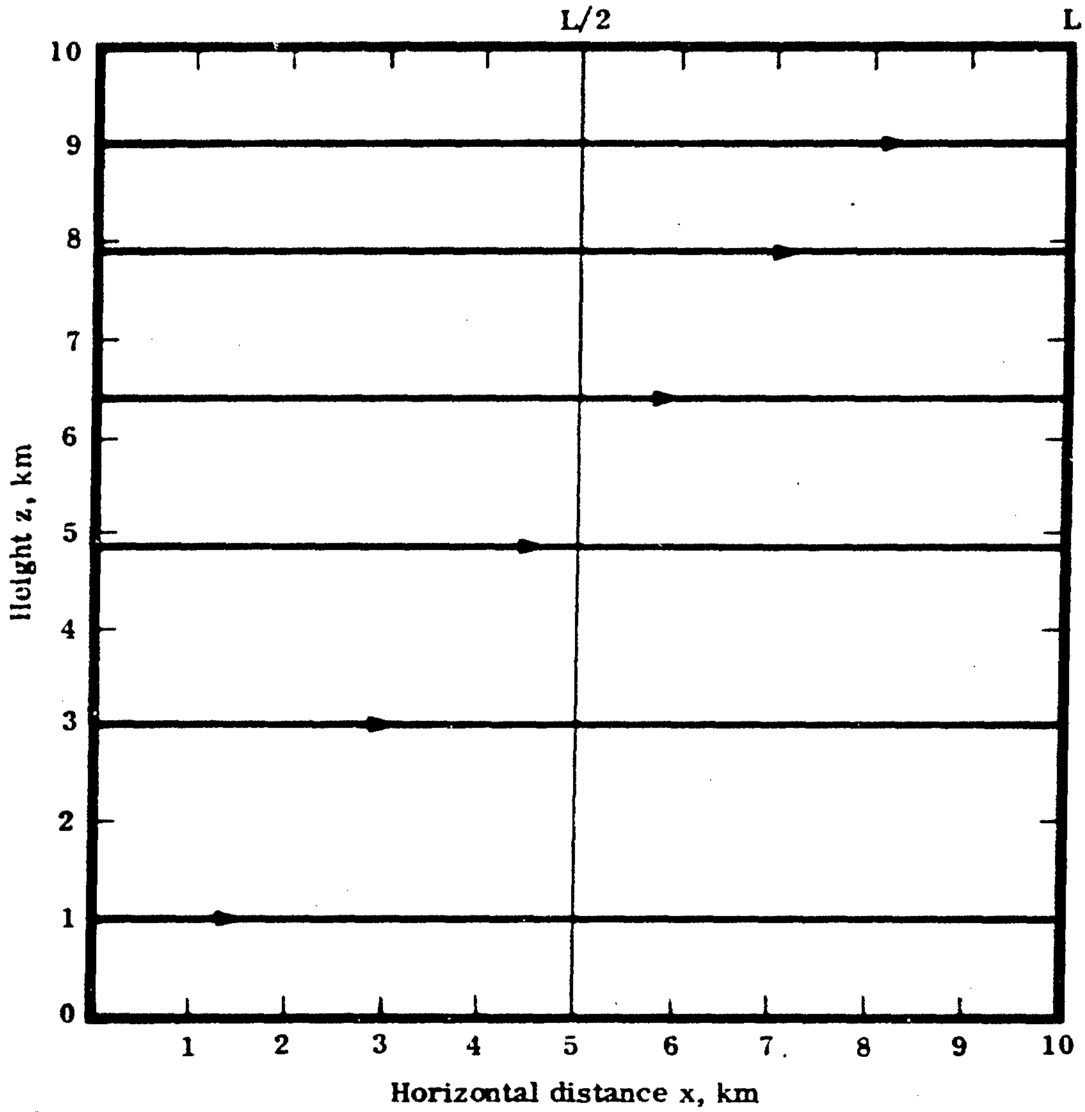

Fig. 5a. Schematic representation of streamlines in a two-dimensional shearing flow without a local circulation.

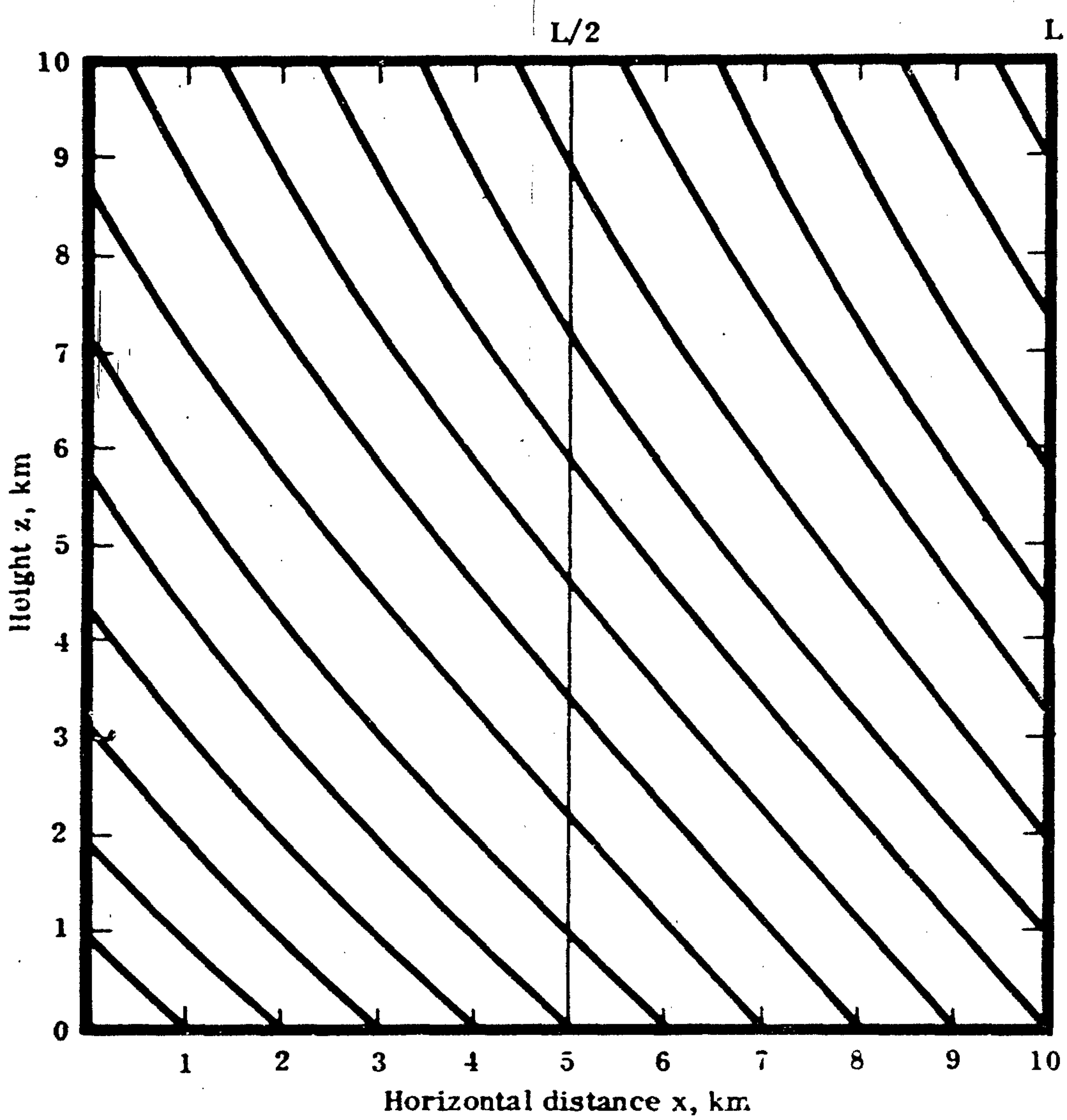

Fig. 5b. Trajectories of fallout descending through the wind field depicted in Fig. 5a.

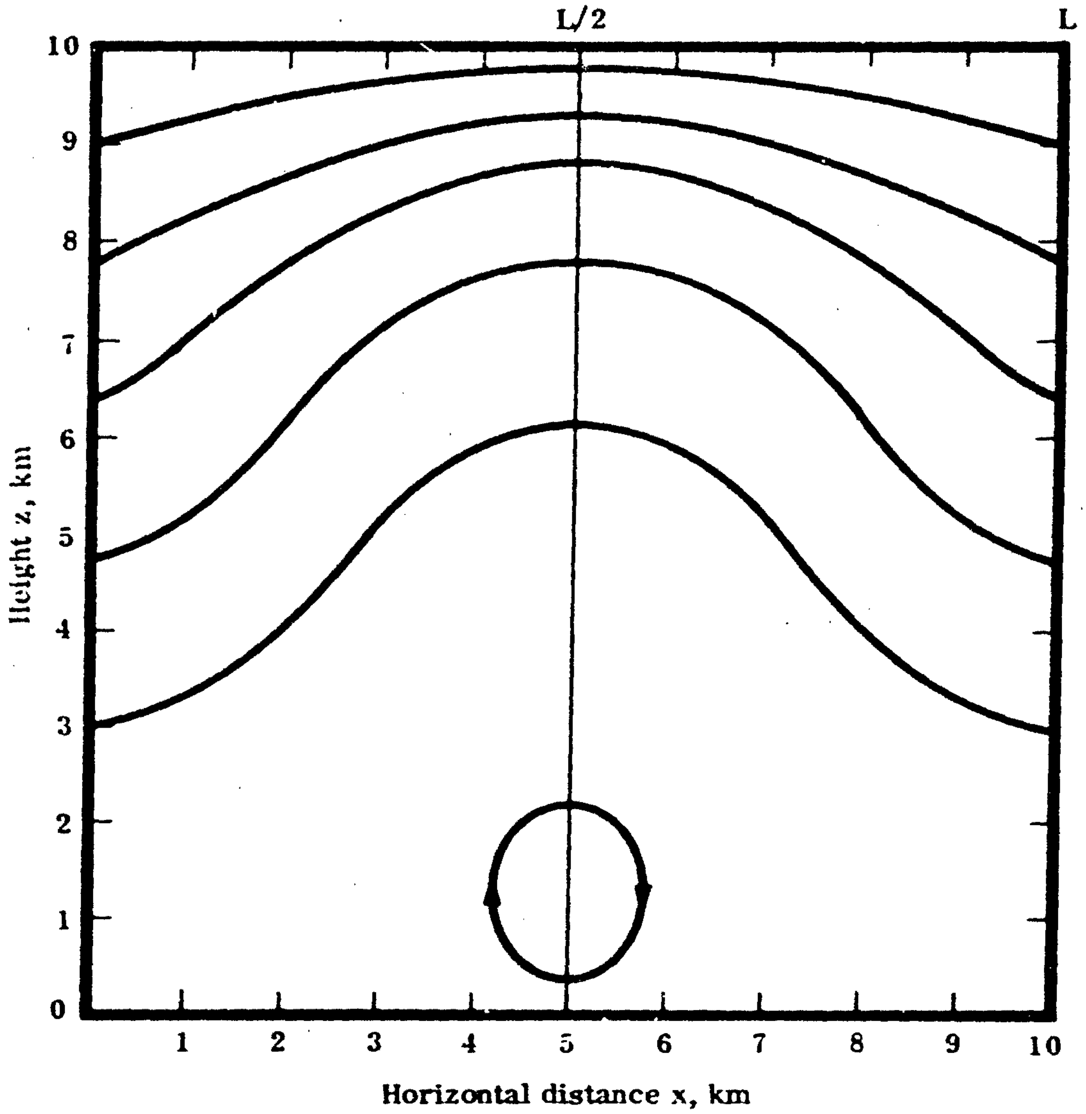

Fig. 6a. Streamlines simulating a kind of airflow over a mountain in an atmosphere with wind shear.

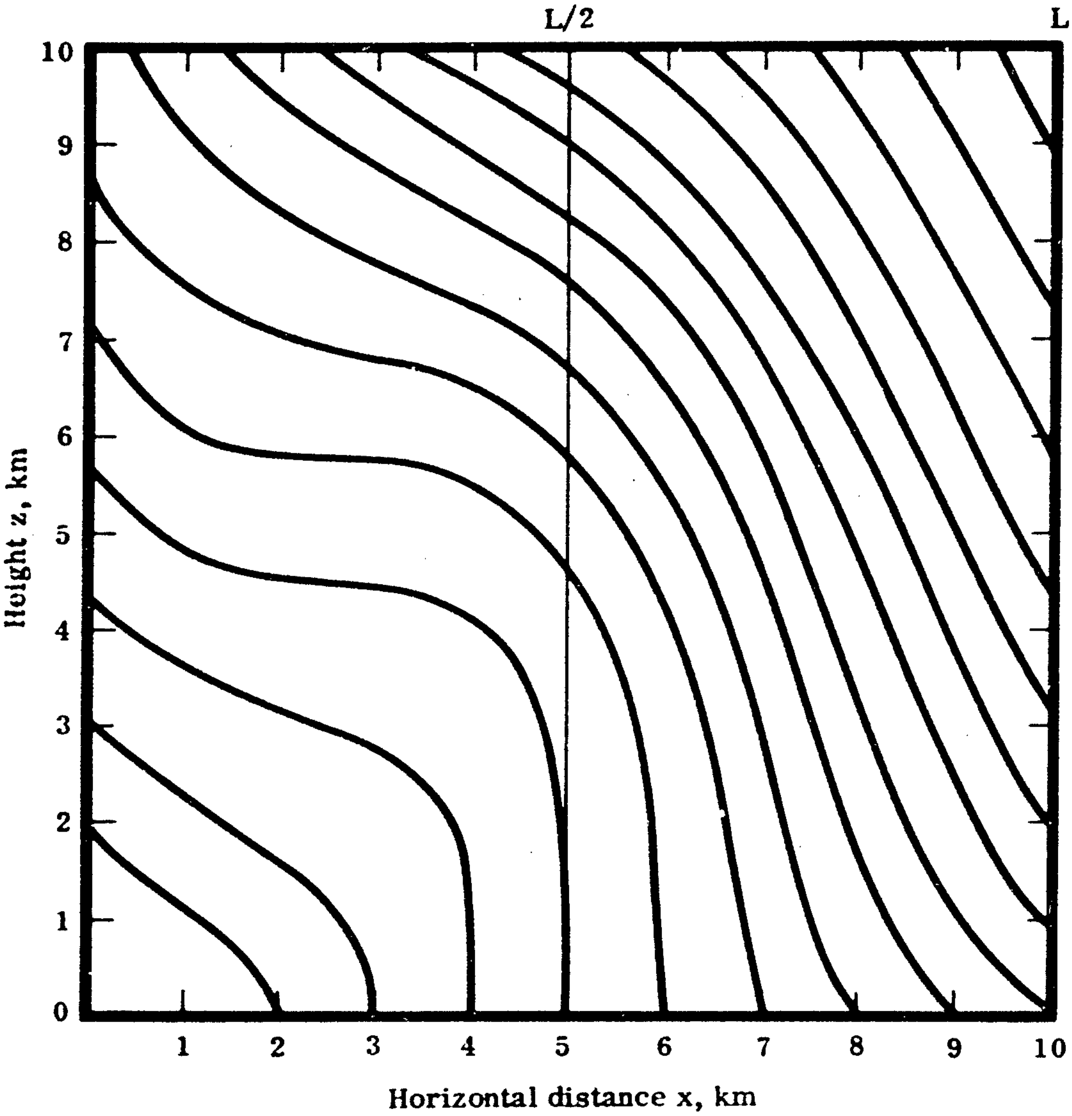

Fig. 6b. Trajectories of fallout descending through the circulation depicted in Fig. 6a.

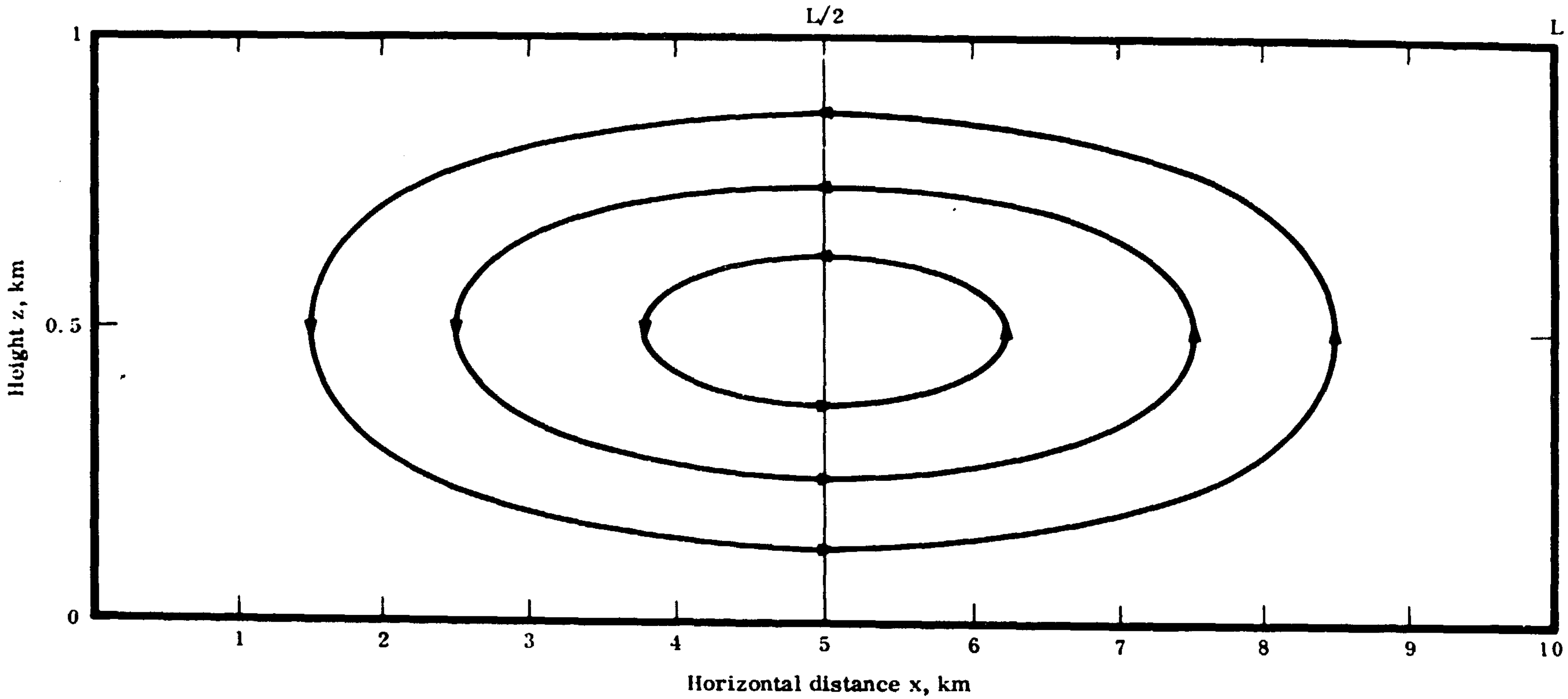

Fig. 7. Local circulation given by the stream function

$$\psi = \frac{2Lw_{max}}{\pi H}\left(z - \frac{z^2}{H}\right)\left(\cos\frac{2\pi x}{L} - 1\right)$$

Also, $\psi(x, z)$ must be constant on the boundary of R because of conditions Eq. (21), i.e., if $(x_0 z_0)$ and $(x_1 z_1)$ are both on the boundary of R, $\psi(x_0 z_0) = \psi(x_1 z_1)$. It follows that the general integral of Eq. (22) is

$$\psi(x, z) + V_i x - \int_{z_0}^{z_1} f(s)\, ds = C_1 . \tag{24}$$

Hence a particle that enters R at the point $(x_0 z_0)$ describes the curve

$$V_i x - \int_{z_0}^{z} f(s)\, ds = V_i x_0 . \tag{25}$$

Equation (25) says that the exit point depends only on the entrance point, the fall speed V_i, and the environmental wind field f(s). In particular, it is independent of the local wind field.

The important implication of the above is that such idealized steady-state circulations as the two-dimensional mountain wave model and the sea breeze parallel to a straight coast have, in the absence of diffusion, only the effect of altering the time of establishment of a fallout pattern, and have no effect on the ultimate shape of the pattern or its concentration. It is shown in Section 1.5 that the variable fall velocity of particles and the compressible nature of the real atmosphere only slightly alter this conclusion.

Such a simple theorem does not hold for a steady axially-symmetric vertical circulation; e.g., one with updrafts in a central region and a surrounding ring of subsiding air which is embedded in a general flow. An example of such a circulation is shown in Fig. 8a. Figure 8b illustrates the flow resulting from the addition of the circulation in Fig. 8a to a uniform flow. A particle at point P in the x - z plane (Fig. 8b) follows the trajectory PQT and will remain in the x - z plane during its descent. A particle arriving at R, however, will not remain in the plane ABCD, (parallel to the x, z plane) since the local circulation carries it to a point S wherefrom it follows the direction of the ambient flow in the x-direction. It therefore does not arrive at that point on the ground that would be its destination if the local circulation were zero. This illustrated circulation thus redistributes the particles. When such circulations are large compared to the extent of fallout, the pattern of fallout may be significantly

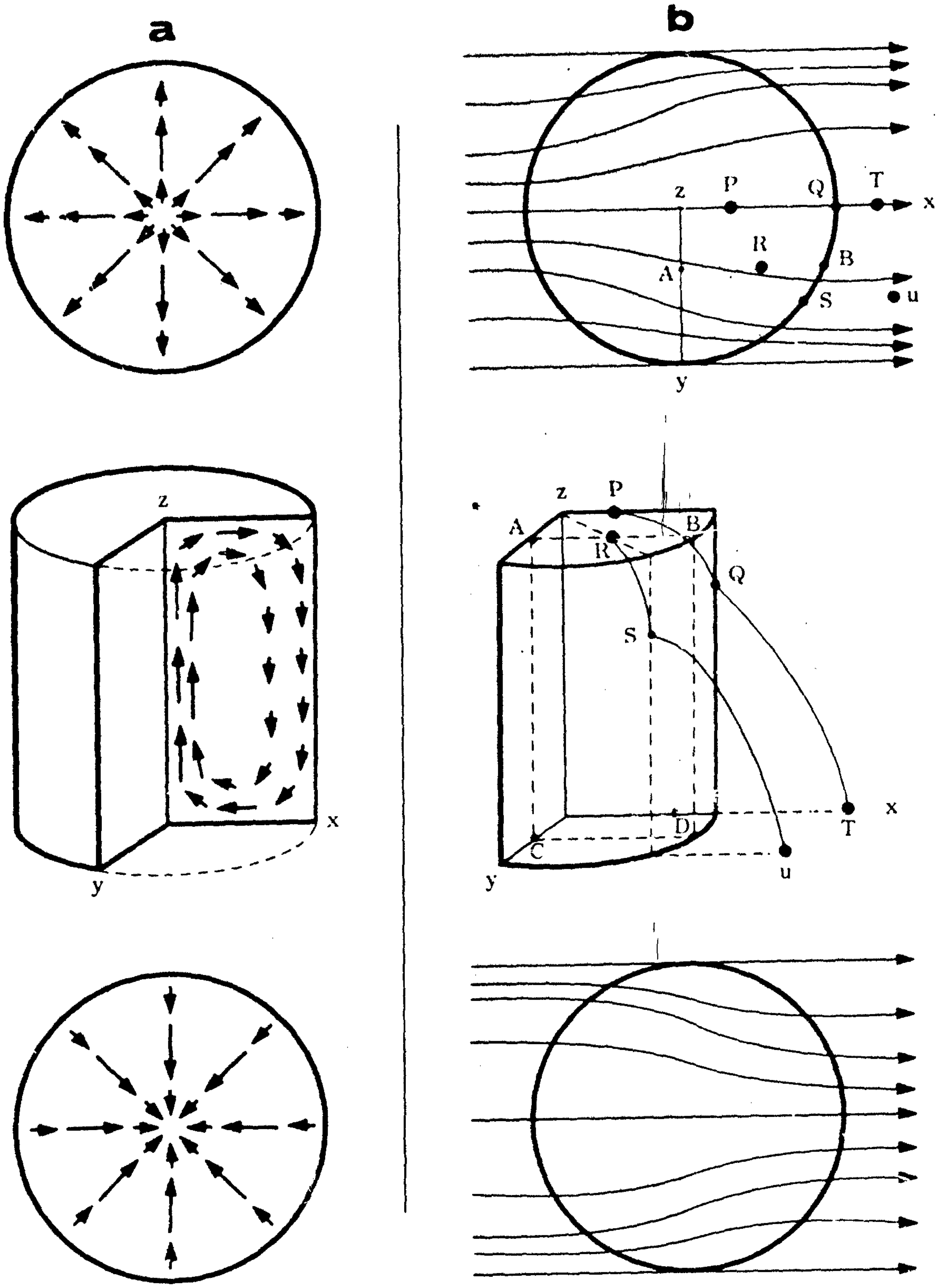

Fig. 8a. Vertical cross-section and plan views of air parcel trajectories near the top and the bottom of a radially symmetric circulation.

Fig. 8b. Plan view of air parcel trajectories near the top and the bottom of the same radially symmetric circulation which is combined with an ambient flow and a three-dimensional model of fallout particle trajectories, starting at points P and R on the upper boundary of the circulation.

deformed, but without changes of magnitude (continuity principles require that a stretching in the y-direction is matched by shrinking in the other directions). In such cases, the expected pattern shape and location should be estimated from trajectory computations.

1.4 The Influence on Fallout of Time-dependent Vertical Currents

Both the time of onset and the duration of fallout at a point on the ground can be influenced by time-dependent vertical circulations. It can be shown that the duration of fallout at a point on the ground is increased beneath intensifying updrafts and weakening downdrafts, and decreased beneath weakening updrafts and intensifying downdrafts. Consider particles falling through an updraft which intensifies with time. The first particle falls through the vertical current most rapidly, and the last most slowly. Therefore, the time interval between the emergence of the first and the last particle is greater than the interval separating their entrance. Consider an intensifying downdraft. The first particle may enter at T_1 and emerge at $T_1 + 15$ minutes. The last particle may enter at T_2 and assisted by the stronger downdraft, emerge at $T_2 + 10$ minutes. The time between emergences at the bottom is 5 minutes less than the interval between entrances at the top. The changed duration of fallout at the ground is reflected linearly in a change of the accumulation of fallout at the ground.

These considerations are further illustrated by Fig. 9 which shows schematic trajectories of particles falling through an intensifying circulation. In the rising currents, a given particle experiences stronger low level horizontal convergence for a longer time than it experiences weaker high level horizontal divergence. Therefore, the points of exit of particles converge at the base of updraft columns. A similar argument, and implications of continuity, show that exit points must diverge beneath intensifying downdrafts. The changed spacing of trajectories at the base of the circulation is associated with a changed vertical distance separating emerging particles.

These considerations are still further illustrated by Fig. 10. An intensifying circulation and fields of convergence and divergence produce a deformation of the fallout pattern. Consistent with Fig. 9, this deformation is associated with the advection of sediment from the downdraft to the updraft regions. In accordance

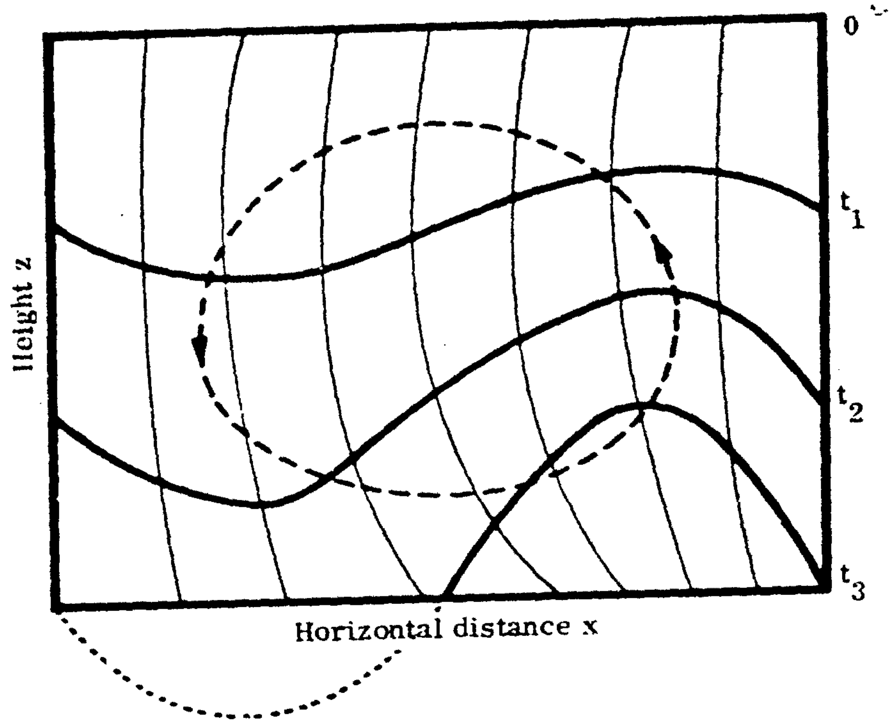

Fig. 9. Schematic indication of trajectories (vertical) and isochrones (horizontal) of fallout which descends through an accelerating local circulation (dashed line).

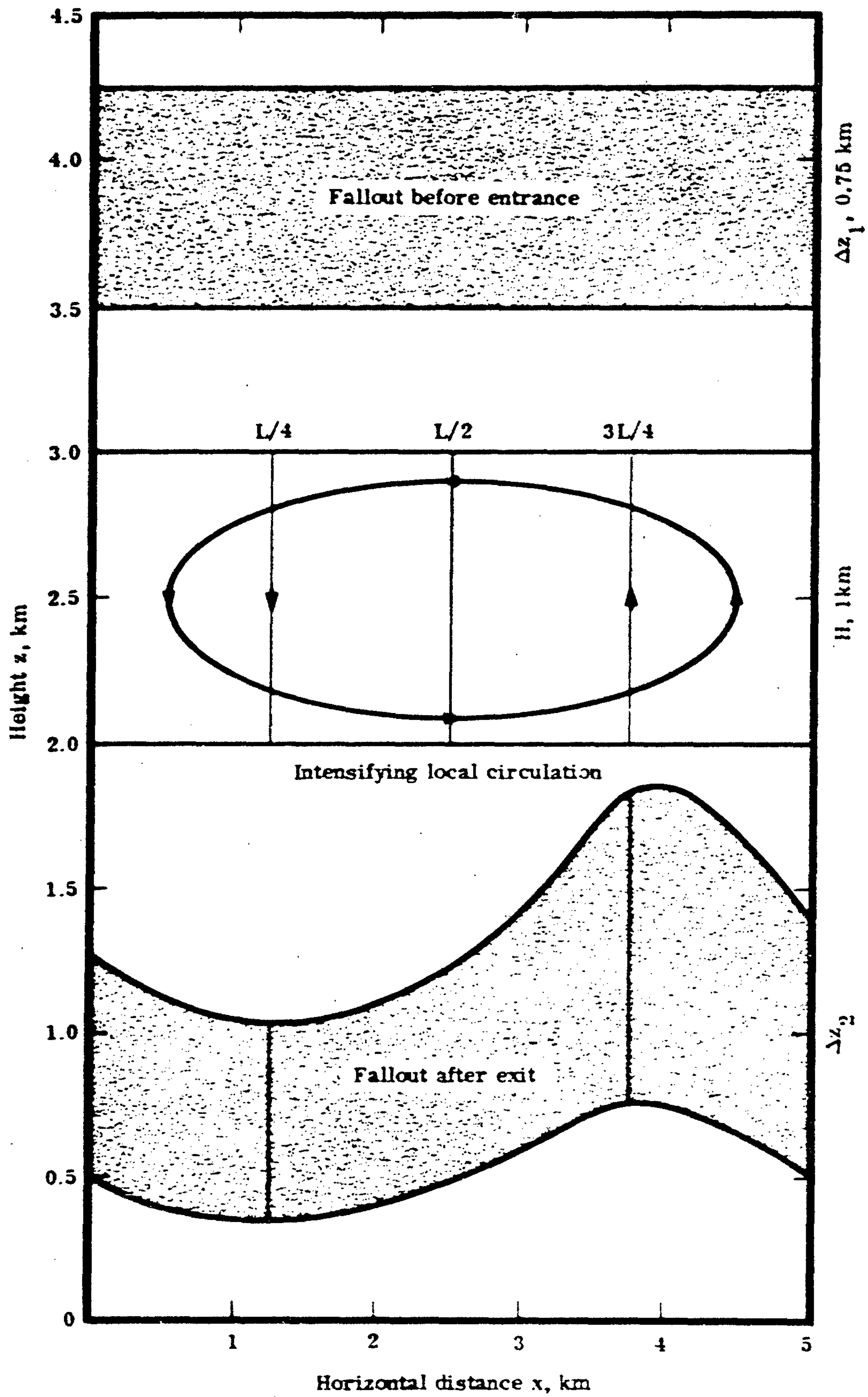

Fig. 10. Deformation of a two-dimensional atomic cloud by an intensifying local circulation.

with Eq. (12), this advection is not associated with changes of concentration in any particle packets, but rather with stretching and shrinking of the vertical extent of the region occupied by particles, and a corresponding change in the duration of fallout at the ground. Figure 11 shows the distribution of onset times and durations of fallout at the ground in this case.

Interesting limiting cases occur when the intensifying circulation attains such strength that the maximum updrafts become as great as the fall speed of particles. Then the duration of fallout beneath the downdraft becomes quite short, while beneath the updraft, the fallout, once started, lasts for an indefinitely long time over an indefinitely small horizontal area at the ground. If maximum updrafts exceed fall speeds before some particles have descended past the point where they can be suspended, the onset of fallout at the ground at the point directly beneath the updraft core will be indefinitely delayed. Of course, these examples serve a pedagogic purpose principally, since circulation changes and diffusion processes will always alter this picture greatly in the real world.

1.5 Compressibility of the Atmosphere in Relation to Fallout

The decrease of particle fall speed and the increase of air density with decreasing height are both shown by Eq. (11) to affect the concentration of fallout. Both terms on the right of Eq. (11) are attributable to the compressibility of the atmosphere.

The total accumulation F_g of fallout on a unit area on the ground may be defined by

$$F_g = - \int_0^{T_g} C_g V_g \, dt \qquad (26)$$

where C_g is the mass of fallout per unit volume of air adjacent to the ground, V_g is the velocity of descent of that fallout, and T_g is the length of time that fallout continues at the ground.

It is readily seen that T_g is unaffected by the presence of a local steady circulation. Consider a region of any depth (H in Fig. 12) through which descends a particle packet of initial depth H_1 as shown in Fig. 12. If successive particles through H have identical velocity histories, the time for the trailing edge of the

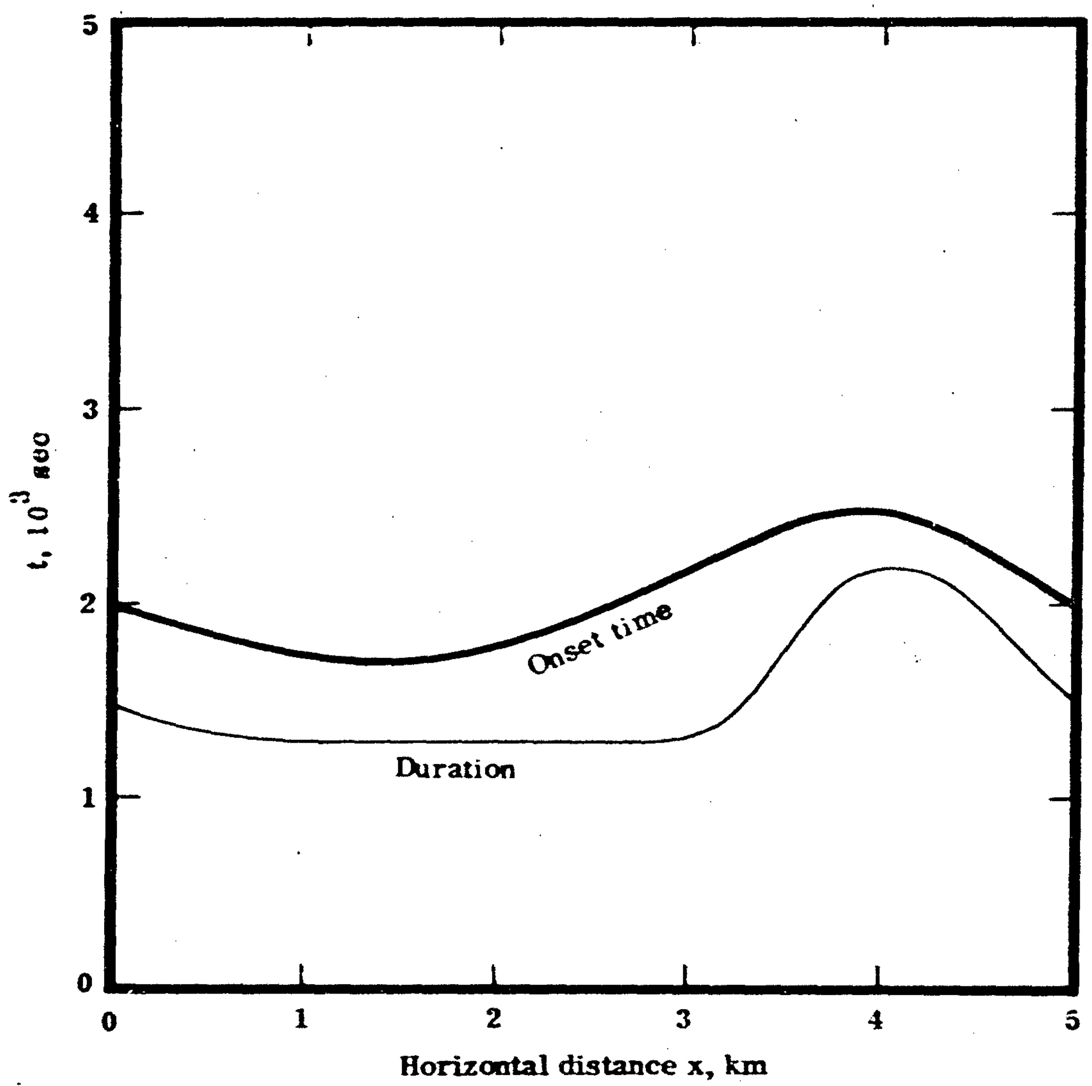

Fig. 11. The onset time (heavy line) and duration (thin line) of fallout at the surface below the intensifying circulation depicted in Fig. 10.

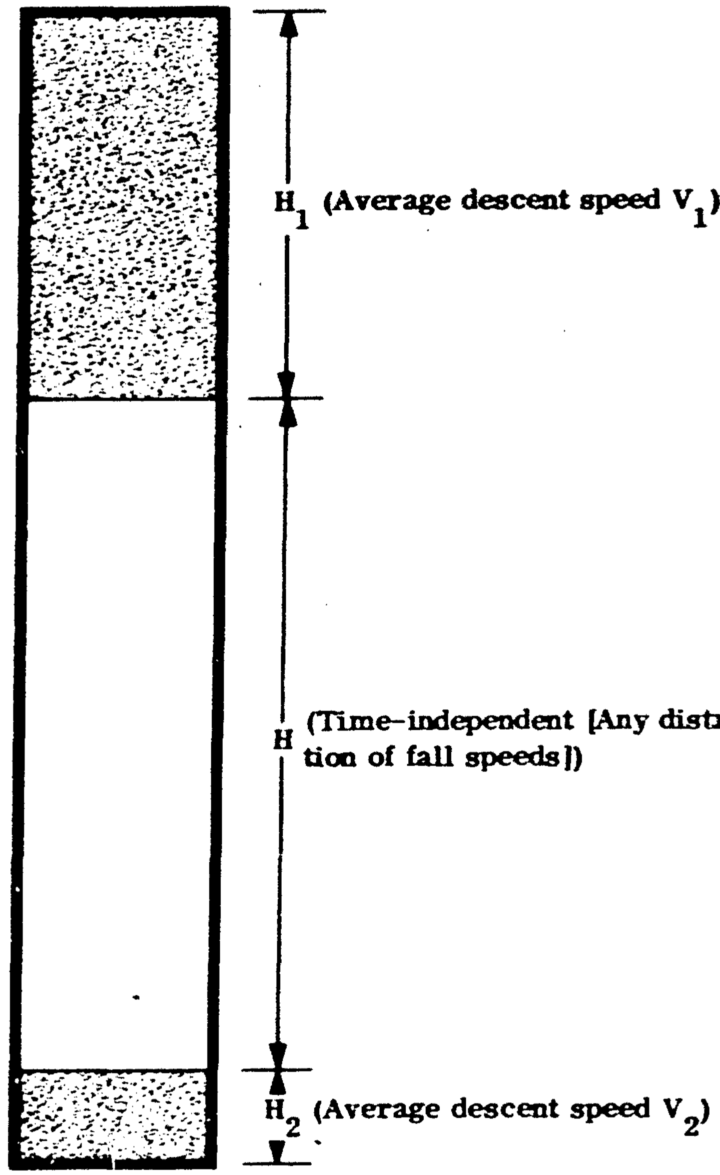

Fig. 12. Vertical shrinkage of a two-dimensional region of fallout due to a decreasing fall velocity of the particles.

column of depth H_1 to arrive at any point is obviously later than the time of arrival of the leading edge, by the interval $-H_1/V_1$. When the packet has descended to a region where the speed of descent is V_2, its height H is evidently reduced to $H_2 = H_1(V_2/V_1)$, for this column, descending at speed V_2, passes a point in the same time as the column of depth H_1, descending at V_1.

The decrease of fall speed following descent and associated shortening of fallout columns, contributes to changed concentrations as defined by Eq. (11). This process alone, however, does not change the accumulation of fallout at the ground, since, in Eq. (26), the increase of C_g would be exactly compensated by decreased V_g.

When a vertical current is present in the column of depth H, other concentration-changing factors can be identified. Since the fall speed decreases with descent, the particle packet may, for example, be longer exposed to horizontal convergence at the base of an updraft than to horizontal divergence at the top. The net effect of the action of horizontal convergence on descending particle packets, however, is reduced in a compressible atmosphere, since the level of no-horizontal divergence is below the height of maximum vertical velocity. In any event, the change of horizontal area of the packet is, by definition,

$$\frac{1}{A}\frac{dA}{dt} = \mathrm{div}_2\,\vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}. \tag{27}$$

The fractional change of cross-sectional area is inversely proportional to total deposit of fallout per unit area, i.e.;

$$\frac{F_2}{F_1} = \frac{A_1}{A_2} = \exp\left[-\int_0^T \mathrm{div}_2\,\vec{V}\,dt\right] = \exp\left[-\int_H^0 \mathrm{div}_2\,\vec{V}\,\frac{dz}{V_i+w}\right] \tag{28}$$

Since $dt = -dz/(V_i+w)$ (positive for downward motions), Eq. (28) can be used to calculate the change of fallout amount when V_i, w, and $\partial \ln \rho/\partial z$ are specified, with $\mathrm{div}_2\,\vec{V}$ defined by Eq. (8a).

Alternatively, Eq. (11) can be used to calculate the concentration at the base of a local circulation; this multiplied by the fall speed there, V_2, is proportional to F_g.

We have

$$\ln \frac{C_1}{C_2} = - \int_0^T \frac{\partial V_i}{\partial z} dt + \int_0^T w \frac{\partial \ln \rho}{\partial z} dt \qquad (29)$$

$$\frac{C_1}{C_2} = \exp \left[\int_H^0 \frac{\partial V_i}{\partial z} \frac{\partial z}{V_i + w} - \int_H^0 w \frac{\partial \ln \rho}{\partial z} \frac{dz}{V_i + w} \right]. \qquad (30)$$

Figures 13 and 14, taken from [3], show variations of fall speed with height for various diameters of fallout particles, and the contribution to the total mass of fallout provided by particles of different sizes. Most of the mass is typically contained in particles about 60 μ, for which

$$V_i = V_0 (1 - az) \text{ m sec}^{-1} \qquad (V_0 = -1 \text{ for } 60\mu \text{ particles})$$
$$(a = 0.25 \times 10^{-4}). \qquad (31)$$

For an example, we take w as defined by Eq. (13) and, in the troposphere,

$$\frac{\partial \ln \rho}{\partial z} = - 10^{-4} \text{ m}^{-1}. \qquad (32)$$

Substitution of Eqs. (31) and (12) in Eq. (30) gives an expression which can be integrated analytically, but this is extremely cumbersome. Much time and labor can be saved by the finite-difference solution given below. Since [see Eqs. (27) and (8)]

$$\frac{1}{A} \frac{dA}{dt} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = - \frac{\partial w}{\partial z} - w \frac{\partial \ln \rho}{\partial z}, \qquad (33)$$

we find, after appropriate substitutions, that

$$\frac{1}{A} \frac{dA}{dz} = \frac{(2z/H) - 1 - 10^{-4}}{V_0 \frac{H(az - 1)}{4w_{max}} + \left(z - \frac{z^2}{H}\right)}. \qquad (34)$$

Solutions of this equation are plotted in Fig. 15.

Figure 15 can be used to estimate the effect of vertical circulations on fallout accumulations attributable to the compressibility of the atmosphere. It should be remembered that infinite concentrations are always confined to an indefinitely small area, that the effect of an updraft to increase the fallout accumulation is mitigated by its corollary action in delaying the arrival of fallout at the surface, and that the calculations are based on steady circulations without diffusion. Of

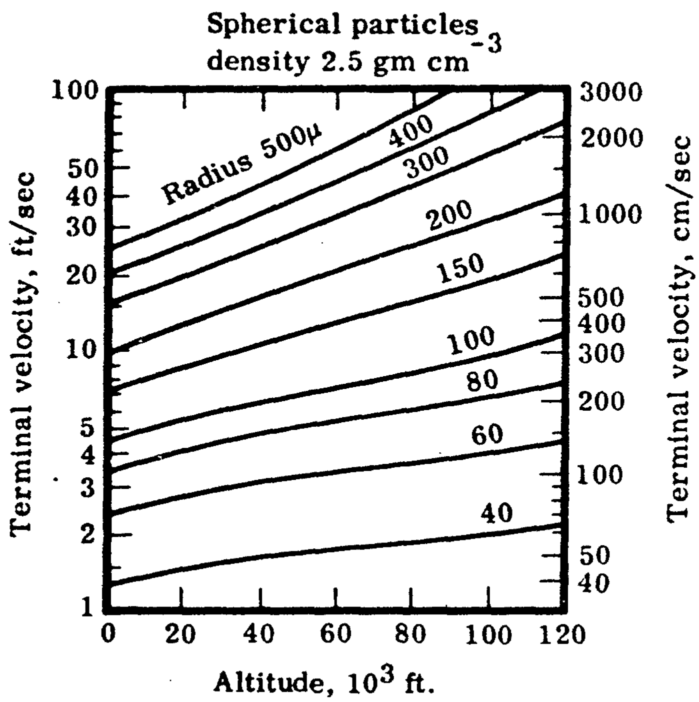

Fig. 13. Terminal velocities of spherical particles as a function of altitude and particle radius.

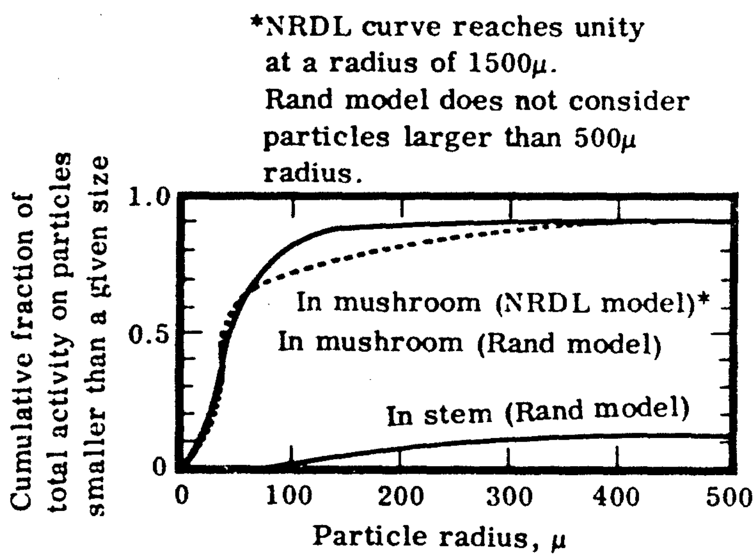

Fig. 14. Size distribution of radioactive particles originating from the mushroom stem of an atomic cloud.

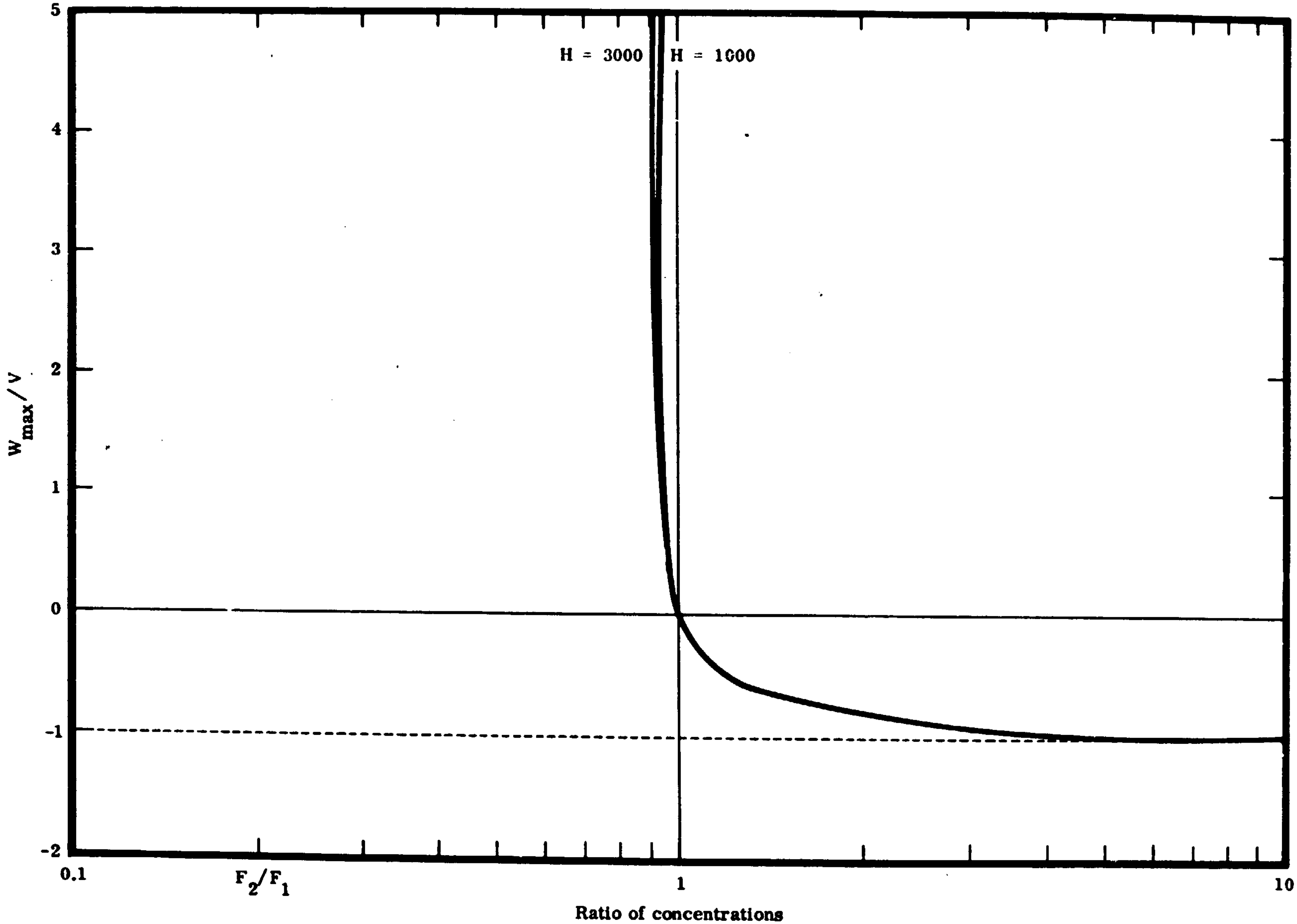

Fig. 15. Ratio of concentration of fallout at the ground associated with updrafts and downdraft columns in a compressible atmosphere. [See Eqs. (13), (31), (32), and (34).]

course, the total amount of fallout is conserved during its descent through the local circulation.

1.6 Local Horizontal Circulations in Relation to Fallout Patterns

Since the time of descent of particles depends on the sum of their terminal fall speed and the vertical velocity of the air, the pattern of fallout which descends through a nondivergent local circulation $(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \frac{\partial w}{\partial z} = w = 0)$ is established in precisely the same time as the pattern without a local wind. However, the local circulation produces a horizontal displacement defined by

$$\Delta X = T \Delta u = - \frac{Z}{\bar{V}} \Delta u, \tag{35}$$

where ΔX is the displacement due to the local circulation; T is the time taken by particles to descend through the circulation of depth Z, Δu is the average difference, at a descending particle, of the actual air velocity and the velocity of the ambient flow, and $\bar{V}$ is the average terminal fall speed. When the average velocity of the ambient flow is relatively small, the displacement due to the local circulation dominates. The importance of the local circulation is proportional to its depth and intensity.

To illustrate the effect of a local horizontal circulation on the pattern of fallout, we have chosen a confluent-diffluent wind field which simulates the flow through a mountain pass. The equations of motion for a confluent-diffluent system are

$$u = u_0 + ax; \qquad v = v_0 - ay \tag{36}$$

in which a is positive for confluence [10]. Each air trajectory is an hyperbola associated with a constant k such that

$$(u_0 + az)(v_0 - ay) = k. \tag{37}$$

In this study, $v_0 = 0$, $u_0 = 10$ m sec^{-1}, and $a_0 = 10^{-3}$ sec^{-1}. The resulting pattern over an example total length of 70 km is shown in Fig. 16.

Figure 17a shows the fallout configuration associated with detonation of a cylindrical atomic cloud 4 km deep and 5 km in diameter, with its base 1 km above the ground. Descent of particles is assumed to occur wholly in the environmental wind field. Note that the lightest particles are most widespread. Figure

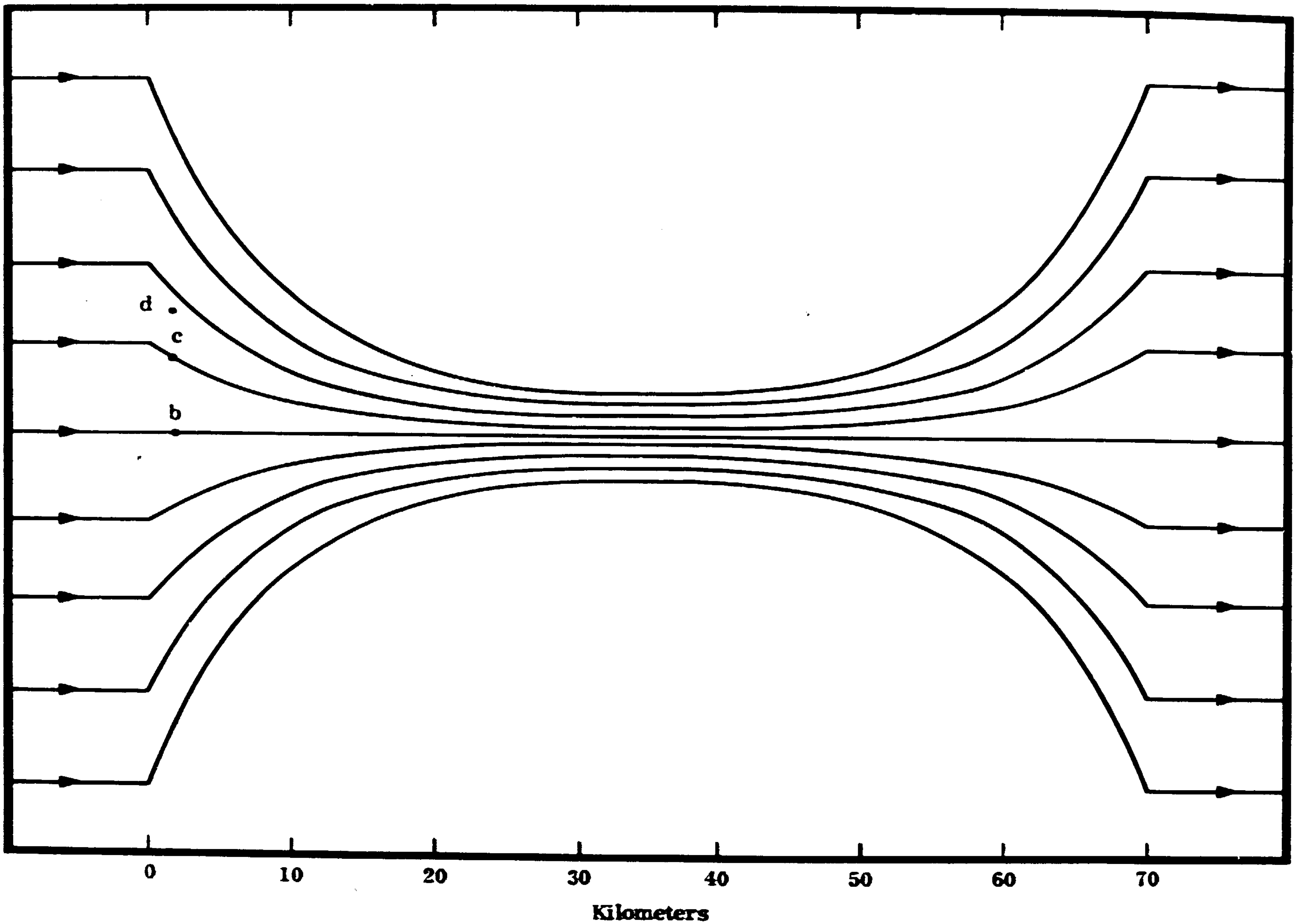

Fig. 16. Confluent-diffluent wind field simulating flow through a mountain pass. The points b, c, and d give the positions of the source cloud corresponding with the fallout patterns B, C, and D in Fig. 17.

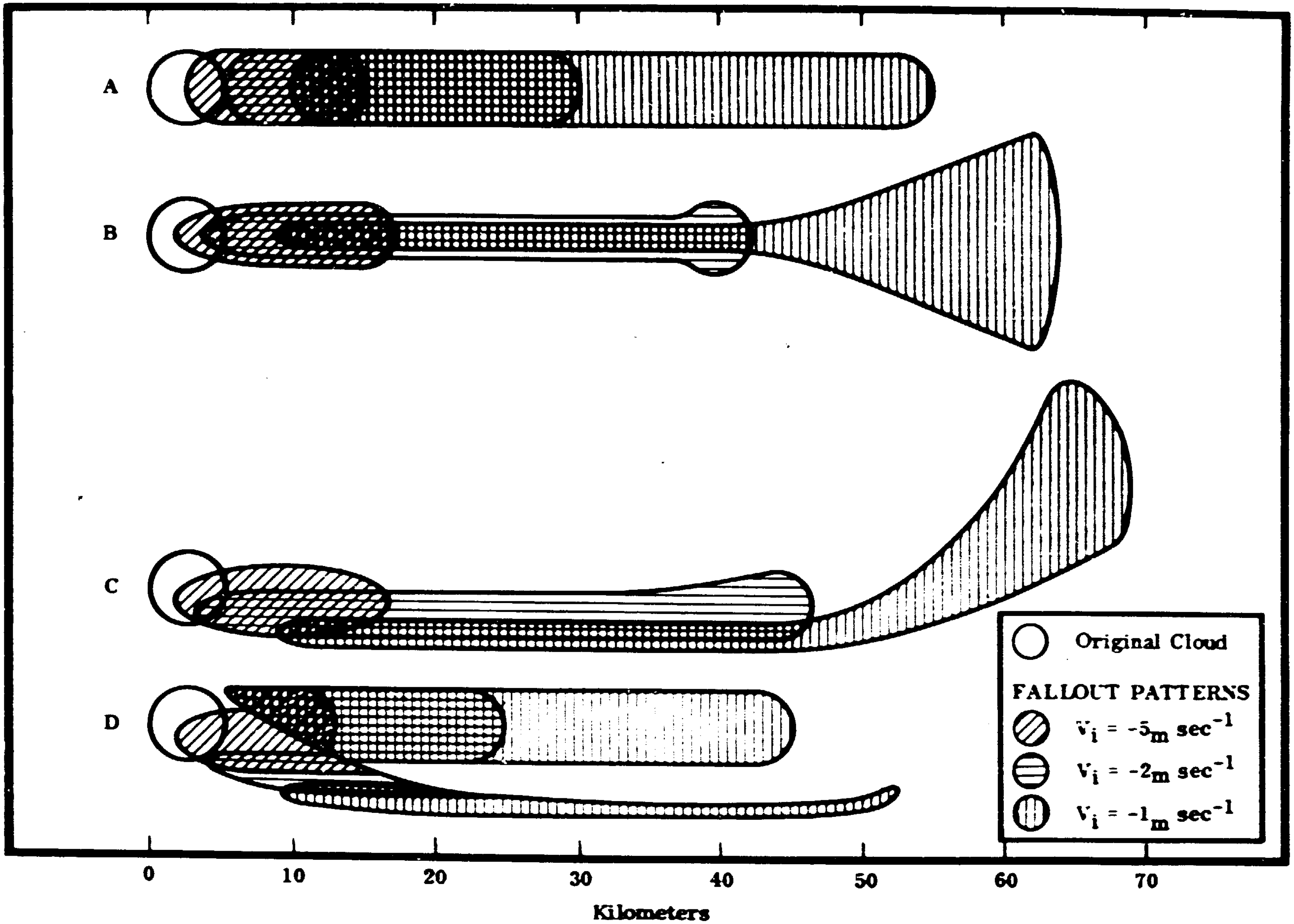

Fig. 17. Patterns of fallout descending from an atomic cloud 5 km deep through a wind field one km deep (shown in Fig. 16) compared with that in undisturbed parallel flow: A. Undisturbed flow; B. Source cloud on axis of symmetry at the entrance of the circulation; C. Source cloud off center; and D. Source cloud at edge of circulation near flat obstacle one km high. Part of the fallout lands on the obstacle.

17b shows the pattern due to an on-axis detonation in the wind field of Fig. 16; detonation off the axis at point c in Fig. 16 is shown in Fig. 17c. Pattern d shows another off-center explosion close to the boundary of an obstacle (mountain) with a flat top at H = 1 km. A part of the material is translated over the top with an undisturbed velocity u and settles. The remainder is caught by the local circulation through which it descends another 1000 m. Note that the influence of the local circulation in all cases is most important with respect to the smallest particles.

2.0 CONCLUSIONS

This study has considered some effects of local circulation on fallout distributions. The role of precipitation processes and diffusion on fallout have been omitted from consideration. Principal conclusions follow.

1. When horizontal steady winds within and without a stabilized atomic cloud are defined, any point on the cloud edge can be associated with a mass-per-unit-area of fallout having a fall speed V_i, a point on the ground, and the times of onset and termination of fallout on the ground.

2. In an incompressible atmosphere, the concentration following the motion of particles with constant fall speed V_i remains constant. In the compressible atmosphere, the local fallout concentrations beneath updrafts increase as the updraft velocity intensifies. Changes of accumulation by a factor of two or more, however, depend on the long continued settling of particles in vertical drafts whose speeds are nearly the same as the terminal fall speeds of the particles. Thus large local increases attributable to compressibility occur only under highly idealized conditions whose occurrence in the real atmosphere should be investigated.

3. When a steady local circulation occurs about a horizontal axis, i.e., when one of the horizontal wind components is always zero, the pattern of fallout eventually established is identical to that produced by the ambient non-divergent flow acting alone. However, horizontal winds accompanying vertical currents which are symmetric about a vertical axis redistribute fallout particles and may change the pattern of their accumulation at the ground. In either case, the time of arrival of particles is delayed by updrafts and expedited by downdrafts.

4. In time-dependent vertical circulations, the duration and total accumulation of fallout with constant fall velocity is increased at a point on the ground beneath intensifying updrafts and decreased beneath weakening updrafts; beneath downdrafts the converse holds. Large changes of fallout concentrations occur in association with small changes of the vertical air currents when their speed is nearly the same as the fall speed of particles. In just these cases neglected diffusion must be most important, since the time for descent of particles is long.

5. The pattern of fallout which descends through a nondivergent local circulation is established in precisely the same time as the pattern without a local wind. The displacement of the pattern and its change of shape by the local circulation is proportional to the circulation's depth and intensity.

6. The influence of the local circulation is, in all cases, most important with respect to the smallest (slow falling) particles.